On $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideals in BF-algebras

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Abstract---- The purpose of this paper is to introduce the concept of $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideal in BF-algebra and investigate some of their related properties, where $\overline{\alpha}$, $\overline{\beta}$ are any one of $\overline{\in}$, $\overline{q}_k$, $\overline{\in} \vee \overline{q}_k$, $\overline{\in} \wedge \overline{q}_k$ unless otherwise specified.

Index terms---- BF-algebra, fuzzy ideal, fuzzy p-ideal, fuzzy subgroups, $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideal, $(\overline{\alpha}, \overline{\beta})$-fuzzy subalgebra, $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$**-fuzzy ideals**

1. Introduction

The notion of BF-algebra was first initiated by Zadeh et al. in his classic paper [26] and investigated some of their properties. The theories were enriched by many authors (see [1, 7, 8, 9, 26]).

The fuzzy set was given by Zadeh in his seminal paper [25], of 1965 provides a natural framework for generalizing some of the basic notions of algebra. Extensive applications of fuzzy set theory have been found in various fields, for example, artificial intelligence, computer science, control engineering, expert system, management science, operation research and many others. The concept was applied to the theory of groupoids and groups by Rosenfeld [19], where he introduced the fuzzy subgroup of a group. Since then the literature of various algebraic structures has been fuzzified.

A new type of fuzzy subgroup, which is, the $(\in, \in \vee q)$-fuzzy subgroup, was introduced by Bhakat and Das [4] by using the combined notions of "belongingness" and "quasi-coincidence" of fuzzy points and fuzzy sets, which was introduced by Pu and Liu [18]. Murali [17] proposed the definition of fuzzy point belonging to a fuzzy subset under natural equivalence on fuzzy subsets. It was found that the most viable generalization of Rosenfeld's fuzzy subgroup is $(\in, \in \vee q)$-fuzzy subgroup. Bhakat [2-3] introduced the concept of ($\in \vee q$)-level subsets, $(\in, \in \vee q)$-fuzzy normal, quasi-normal and maximal subgroups. Many researchers utilized these concepts to generalize some concepts of algebra (see [5-6, 10-13, 24, 28-30]). Davvaz in [6] discussed $(\in, \in \vee q)$-fuzzy sub-near-rings and ideals. In [10-12], Jun defined the concept of (α, β)-fuzzy sub-algebras/ideals in BCK/BCI-algebras where α, β are any of {$\in$, q, $\in \vee q$, $\in \wedge q$} with $\alpha \neq \in \wedge q$. Zulfiqar introduced the notion of (α, β)-fuzzy positive implicative ideals in BCK-algebras [28]. Generalizing the concept of quasi-coincident of a fuzzy point with a fuzzy set, in [13], Jun defined $(\in, \in \vee q_k)$-fuzzy sub-algebras in BCK/BCI-algebras. In [14], Jun et al. discussed $(\in, \in \vee q_k)$-fuzzy ideals in BCK/BCI-algebras. Khan et al. studied order semi-groups characterized by $(\in, \in \vee q_k)$-fuzzy generalized bi-ideals [15]. Larimi [16], initiated the notion of $(\in, \in \vee q_k)$-intuitionistic fuzzy ideals of hemirings. Shabir et al. [20], characterized different classes of semi-groups by $(\in, \in \vee q_k)$-fuzzy ideals and $(\in, \in \vee q_k)$-fuzzy bi-ideals. Shabir and Rehman [21], defined ternary semi-groups by $(\in, \in \vee q_k)$-fuzzy ideals. Tang and Xie, studied $(\in, \in \vee q_k)$-Fuzzy ideals of ordered semi-groups [23]. Shabir and Mahmood [22], defined the concept of semi-hyper-groups characterized by $(\in, \in \vee q_k)$-fuzzy hyper-ideals. In [27], Zeb et al. studied the characterization of ternary semi-groups in terms of $(\in, \in \vee q_k)$-ideals. The notion of $(\in, \in \vee q_k)$-fuzzy fantastic ideals in BCI-algebras was introduced by Zulfiqar in [30] and investigated some of their related properties.

In the present paper, we define the notion of $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideal in BF-algebra and investigated some of their related properties, where $\overline{\alpha}$, $\overline{\beta}$ are any one of $\overline{\in}$, $\overline{q}_k$, $\overline{\in} \vee \overline{q}_k$, $\overline{\in} \wedge \overline{q}_k$ unless otherwise specified.

2. Preliminaries

Throughout this paper κ always denote a BF-algebra without any specification. We also include some basic aspects that are necessary for this paper.

By a BF-algebra [1, 26], is a general algebra $(\kappa, *, 0)$ of type (2, 0) satisfying the following conditions:

(BF-1) $d * d = 0$
(BF-2) $d * 0 = d$
(BF-3) $0 * (d * e) = (e * d)$
for all $d, e \in \kappa$.

We can define a partial order "$\leq$" on κ by $d \leq e$ if and only if $d * e = 0$.

Proposition 2.1. [1, 26] In any BF-algebra κ, the following are true:

(i) $0 * (0 * d) = d$
(ii) $0 * d = 0 * e$, then $d = e$
(iii) $d * e = 0$, then $e * d = 0$
(iv) $d * 0 = d$
for all $d, e \in \kappa$.

Definition 2.2. [7] A nonempty subset S of a BF-algebra κ is called a sub-algebra of κ if it satisfies

$d * e \in S$, for all $d, e \in S$.

Definition 2.3. [7] A non-empty subset I of a BF-algebra κ is called an ideal of κ if it satisfies the conditions (I1) and (I2), where
(I1) $0 \in I$,
(I2) $d * e \in I$ and $e \in I$ imply $d \in I$,
for all $d, e \in \kappa$.

Definition 2.4. A non-empty subset I of a BF-algebra κ is called a fuzzy p-ideal of κ if it satisfies the conditions (I1) and (I3), where
(I1) $0 \in I$,
(I3) $(d * f) * (e * f) \in I, e \in I \Rightarrow d \in I$,
for all $d, e, f \in \kappa$.

Definition 2.5. [7] A fuzzy set Υ of a BF-algebra κ is called a fuzzy ideal of κ if it satisfies the conditions (F1) and (F2), where
(F1) $\Upsilon(0) \geq \Upsilon(d)$,
(F2) $\Upsilon(d) \geq \Upsilon(d * e) \wedge \Upsilon(e)$,
for all $d, e \in \kappa$.

Definition 2.6. A fuzzy set Υ of a BF-algebra κ is called a fuzzy p-ideal of κ if it satisfies the conditions (F1) and (F3), where
(F1) $\Upsilon(0) \geq \Upsilon(d)$,
(F3) $\Upsilon(d) \geq \Upsilon((d * f) * (e * f)) \wedge \Upsilon(e)$,
for all $d, e, f \in \kappa$.

Theorem 2.7. Let $\{\Upsilon_i \mid i \in I\}$ be a family of fuzzy p ideals of a BF-algebra κ. Then $\Upsilon = \bigvee_{i \in I} \Upsilon_i$ is a fuzzy p-ideal of κ.

Proof. Let $\{\Upsilon_i \mid i \in I\}$ be a family of fuzzy p ideals of a BF-algebra κ and $d \in \kappa$. Since

$$\Upsilon_i(0) \geq \Upsilon_i(d)$$

for all $i \in I$, we have

$$\begin{aligned}(\bigvee_{i \in I} \Upsilon_i)(0) &= \bigvee_{i \in I} (\Upsilon_i(0)) \\ &\geq \bigvee_{i \in I} (\Upsilon_i(d)) \\ &= (\bigvee_{i \in I} \Upsilon_i)(d).\end{aligned}$$

Thus

$$(\bigvee_{i \in I} \Upsilon_i)(0) \geq (\bigvee_{i \in I} \Upsilon_i)(d).$$

Let $d, e, f \in \kappa$. Since each Υ_i is a fuzzy p-ideal of κ. So

$$\Upsilon_i(d) \geq \Upsilon_i((d * f)) * (e * f)) \wedge \Upsilon_i(e)$$

for all $i \in I$. Thus

$$\begin{aligned}(\bigvee_{i \in I} \Upsilon_i)(d) &= \bigvee_{i \in I} (\Upsilon_i(d) \\ &\geq \bigvee_{i \in I} (\Upsilon_i((d * f) * (e * f)) \wedge \Upsilon_i(e)) \\ &= (\bigvee_{i \in I} \Upsilon_i)((d * f)) * (e * f)) \wedge (\bigvee_{i \in I} \Upsilon_i)(e)\end{aligned}$$

Thus

$$(\bigvee_{i \in I} \Upsilon_i)(d) \geq (\bigvee_{i \in I} \Upsilon_i)((d * f)) * (e * f)) \wedge (\bigvee_{i \in I} \Upsilon_i)(e)).$$

Hence, $\bigvee_{i \in I} \Upsilon_i$ is a fuzzy p-ideal of κ.

A fuzzy set Υ of a BF-algebra κ having the form

$$\Upsilon(e) = \begin{cases} t \in (0, 1] & \text{if } e = d, \\ 0 & \text{if } e \neq d, \end{cases}$$

is said to be a fuzzy point with support d and value t and is denoted by d_t.

For a fuzzy point d_t and a fuzzy set Υ in a set κ, Pu and Liu [18] gave meaning to the symbol $d_t \alpha \Upsilon$, where $\alpha \in \{\in, q, \in \vee q, \in \wedge q\}$. A fuzzy point d_t

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is said to belong to (resp., quasi-coincident with) a fuzzy set Υ, written as $d_t \in Υ$ (resp. $d_t q\, Υ$) if $Υ(d) \geq t$ (resp. $Υ(d) + t > 1$). By $d_t \in \vee\, qΥ$ ($d_t \in \wedge\, qΥ$) we mean that $d_t \in Υ$ or $d_t qΥ$ ($d_t \in Υ$ and $d_t qΥ$). For all $t_1, t_2 \in [0, 1]$, $\min\{t_1, t_2\}$ and $\max\{t_1, t_2\}$ will be denoted by $t_1 \wedge t_2$ and $t_1 \vee t_2$, respectively.

In what follows let α and β denote any one of $\in$, q, $\in \vee$ q, $\in \wedge$ q and $\alpha \neq \in \wedge$ q unless otherwise specified. To say that $d_t \overline{\alpha}\, Υ$ means that $d_t\, \alpha\, Υ$ does not hold.

3. $(\overline{\alpha}, \overline{\beta})$-fuzzy ideals

In this section, we introduce the concept of $(\overline{\alpha}, \overline{\beta})$-fuzzy ideals in a BF-algebra, where $\overline{\alpha}$, $\overline{\beta}$ are any one of $\overline{\in}$, $\overline{q}_k$, $\overline{\in} \vee \overline{q}_k$, $\overline{\in} \wedge \overline{q}_k$ unless otherwise specified.

Definition 3.1. A fuzzy subset Υ of a BF-algebra κ is called an $(\overline{\alpha}, \overline{\beta})$-fuzzy subalgebra of κ, where $\overline{\alpha} \neq \overline{\in} \wedge \overline{q}_k$, if it satisfies the condition

$$(d * e)_{t_1 \wedge t_2}\, \overline{\alpha}\, Υ \Rightarrow d_{t_1}\, \overline{\beta}\, Υ \text{ or } e_{t_2}\, \overline{\beta}\, Υ,$$

for all $t_1, t_2 \in (0, 1]$ and $d, e \in κ$.

Definition 3.2. A fuzzy subset Υ of a BF-algebra κ is called an $(\overline{\alpha}, \overline{\beta})$-fuzzy ideal of κ, where $\overline{\alpha} \neq \overline{\in} \wedge \overline{q}_k$, if it satisfies the conditions (A) and (B), where

(A) $0_t\, \overline{\alpha}\, Υ \Rightarrow d_t\, \overline{\beta}\, Υ$,

(B) $d_{t_1 \wedge t_2}\, \overline{\alpha}\, Υ \Rightarrow (d * e)_{t_1}\, \overline{\beta}\, Υ$ or $e_{t_2}\, \overline{\beta}\, Υ$,

for all $t, t_1, t_2 \in (0, 1]$ and $d, e \in κ$.

Definition 3.3. A fuzzy subset Υ of a BF-algebra κ is called an $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideal of κ, where $\overline{\alpha} \neq \overline{\in} \wedge \overline{q}_k$, if it satisfies the conditions (A) and (C), where

(A) $0_t\, \overline{\alpha}\, Υ \Rightarrow d_t\, \overline{\beta}\, Υ$,

(C) $d_{t_1 \wedge t_2}\, \overline{\alpha}\, Υ \Rightarrow ((d * f) * (e * f))_{t_1}\, \overline{\beta}\, Υ$ or $e_{t_2}\, \overline{\beta}\, Υ$,

for all $t, t_1, t_2 \in (0, 1]$ and $d, e, f \in κ$.

Theorem 3.4. Every $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideal of a BF-algebra κ is an $(\overline{\alpha}, \overline{\beta})$-fuzzy ideal of κ.

Proof. Let Υ be an $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideal of κ. Then for all $t_1, t_2 \in (0, 1]$ and $d, e, f \in κ$, we have

$$d_{t_1 \wedge t_2}\, \overline{\alpha}\, Υ \Rightarrow ((d * f) * (e * f))_{t_1}\, \overline{\beta}\, Υ \text{ or } e_{t_2}\, \overline{\beta}\, Υ.$$

Putting f = 0 in above, we get

$$d_{t_1 \wedge t_2}\, \overline{\alpha}\, Υ \Rightarrow ((d * 0) * (e * 0))_{t_1}\, \overline{\beta}\, Υ \text{ or } e_{t_2}\, \overline{\beta}\, Υ$$

$$d_{t_1 \wedge t_2}\, \overline{\alpha}\, Υ \Rightarrow (d * e)_{t_1}\, \overline{\beta}\, Υ \text{ or } e_{t_2}\, \overline{\beta}\, Υ$$

$$d_{t_1 \wedge t_2}\, \overline{\alpha}\, Υ \Rightarrow (d * e)_{t_1}\, \overline{\beta}\, Υ \text{ or } e_{t_2}\, \overline{\beta}\, Υ$$

(by BF-2)

This means that Υ satisfies the condition (B). Combining with (A) implies that Υ is an $(\overline{\alpha}, \overline{\beta})$-fuzzy ideal of κ.

4. $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy ideals

In this section, we introduce the concept of $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy ideas in a BF-algebra and investigate some of their properties.

Consider the number $t \in (\frac{1-k}{2}, 1]$ for which $Υ_t$ is a p-ideal of κ, we consider a new kind of a fuzzy p-ideal as follows.

Definition 4.1. Let Υ be a fuzzy subset of a BF-algebra κ. Then Υ is called an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy ideal of κ if it satisfies the conditions (D) and (E), where

(D) $0_t\, \overline{\in}\, Υ \Rightarrow d_t\, \overline{\in} \vee \overline{q}_k Υ$,

(E) $d_{t \wedge r}\, \overline{\in}\, Υ \Rightarrow (d * e)_t\, \overline{\in} \vee \overline{q}_k Υ$ or $e_r\, \overline{\in} \vee \overline{q}_k Υ$,

for all $d, e \in κ$ and $t, r \in (0, 1]$.

Theorem 4.2. The conditions (D) and (E) in Definition 4.1, are equivalent to the following conditions, respectively:

(F) $Υ(0) \vee \frac{1-k}{2} \geq Υ(d)$,

(G) $Υ(d) \vee \frac{1-k}{2} \geq Υ(d * e) \wedge Υ(e)$,

for all $d, e \in κ$.

Proof. Straightforward.

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Definition 4.3. Let Υ be a fuzzy subset of a BF-algebra κ. Then Υ is called an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of κ if it satisfies the conditions (D) and (H), where

(D) $\quad 0_t \overline{\in} \Upsilon \;\Rightarrow\; d_t \overline{\in} \vee \overline{q}_k \Upsilon$,

(H) $\quad (d))_{t \wedge r} \overline{\in} \Upsilon \;\Rightarrow\; ((d * f) * (e * f))_t \overline{\in} \vee \overline{q}_k \Upsilon$ or $e_r \overline{\in} \vee \overline{q}_k \Upsilon$,

for all d, e, f $\in \kappa$ and t, r $\in (0, 1]$.

Example 4.4 Assume G = {0, 1, 2, 3} be a BF-algebra with the next Cayley table [5]:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	0
2	2	3	0	2
3	3	0	2	0

Assume Υ be a Fuzzy set in G specified by $\Upsilon(0) = 0.39$, $\Upsilon(1) = \Upsilon\varrho(2) = 0.34$ and $\Upsilon(3) = 0.29$. Then Υ is an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of G for k = 0.2

Theorem 4.5. The conditions (D) and (H) in Definition 4.3, are equivalent to the following conditions, respectively:

(F) $\quad \Upsilon(0) \vee \frac{1-k}{2} \geq \Upsilon(d)$,

(I) $\quad \Upsilon(d) \vee \frac{1-k}{2} \geq \Upsilon((d * f) * (e * f)) \wedge \Upsilon(e)$,

for all d, e, f $\in \kappa$.

Proof. (D) $\Leftrightarrow$ (F)

The proof is obvious.

(H) $\Rightarrow$ (I)

Suppose there exist d, e, f $\in \kappa$ such that

$$\Upsilon(d) \vee \frac{1-k}{2} < t = \Upsilon((d * f) * (e * f)) \wedge \Upsilon(e).$$

Then

$$t \in \left(\frac{1-k}{2}, 1\right],\ (d)_t \overline{\in} \Upsilon \text{ and } ((d * f) * (e * f))_t \in \Upsilon,$$

$e_t \in \Upsilon$.

It follows that

$$((d * f) * (e * f))_t \overline{q}_k \Upsilon \text{ or } e_t \overline{q}_k \Upsilon.$$

Then

$$\Upsilon((d * f) * (e * f)) + t + k \leq 1$$

or

$$\Upsilon(f) + t + k \leq 1.$$

As $t \leq \Upsilon((d * f) * (e * f))$ and $t \leq \Upsilon(f)$, it follows that

$$t \leq \frac{1-k}{2}.$$

This is a contradiction. So

$$\Upsilon(d)) \vee \frac{1-k}{2} \geq \Upsilon((d * f) * (e * f)) \wedge \Upsilon(e).$$

(I) $\Rightarrow$ (H)

Let d, e, f $\in \kappa$ and t, r $\in (0, 1]$ be such that

$$(d)_{t \wedge r} \overline{\in} \Upsilon.$$

Then

$$\Upsilon(d)) < t \wedge r.$$

(*a*) $\quad$ If $\Upsilon(d)) \geq \frac{1-k}{2}$, then by (I), we have

$$\Upsilon(d)) \geq \Upsilon((d * f) * (e * f)) \wedge \Upsilon(e).$$

Thus

$$\Upsilon((d * f) * (e * f)) \wedge \Upsilon(e) < t \wedge r,$$

so

$$\Upsilon((d * f) * (e * f)) < t \text{ or } \Upsilon(e) < r.$$

It follows that

$$((d * f) * (e * f)_t \overline{\in} \Upsilon \text{ or } e_r \overline{\in} \Upsilon,$$

therefore

$$((d * f) * (e * f))_t \overline{\in} \vee \overline{q}_k \Upsilon \text{ or } e_r \overline{\in} \vee \overline{q}_k \Upsilon.$$

(b) $\quad$ If $\Upsilon(d)) < \frac{1-k}{2}$, then by (I)

$$\frac{1-k}{2} \geq \Upsilon((d * f) * (e * f)) \wedge \Upsilon(e).$$

Let $((d * f) * (e * f))_t \in \Upsilon$ and $e_r \in \Upsilon$. Then

$$\Upsilon((d * f) * (e * f)) \geq t \text{ and } \Upsilon(e) \geq r.$$

Thus

$$\frac{1-k}{2} \geq t \wedge r.$$

Hence $\Upsilon((d * f) * (e * f)) \wedge \Upsilon(e) + t \wedge r + k$

$$\leq \frac{1-k}{2} + \frac{1-k}{2} + k = 1,$$

i.e.,

$$((d * f) * (e * f))_t \overline{q}_k \Upsilon \text{ or } e_r \overline{q}_k \Upsilon.$$

Therefore

$$((d * f) * (e * f))_t \overline{\in} \vee \overline{q}_k \Upsilon \text{ or } e_r \overline{\in} \vee \overline{q}_k \Upsilon.$$

Theorem 4.6. Every $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of a BF-algebra κ is an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy ideal of κ.

Proof. The proof follows from Theorem 3.4.

Theorem 4.7. The intersection of any family of $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p ideals of a BF-algebra κ is an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of κ.

Proof. Let $\{\Upsilon_i\}_{i \in I}$ be a family of $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p ideals of a BF-algebra κ and $d \in \kappa$. Since

$$\Upsilon_i(0) \vee \frac{1-k}{2} \geq \Upsilon_i(d)$$

for all $i \in I$, we have

$$\Big(\bigwedge_{i \in I} \Upsilon_i\Big)(0) \vee \frac{1-k}{2} = \bigwedge_{i \in I} \Upsilon_i(0) \vee \frac{1-k}{2} \geq \bigwedge_{i \in I} (\Upsilon_i(d)) = \Big(\bigwedge_{i \in I} \Upsilon_i\Big)(d).$$

Thus

$$\Big(\bigwedge_{i \in I} \Upsilon_i\Big)(0) \vee \frac{1-k}{2} \geq \Big(\bigwedge_{i \in I} \Upsilon_i\Big)(d).$$

Let $d, e \in \kappa$. Since each Υ_i is an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of κ. So

$$\Upsilon_i(d) \vee \frac{1-k}{2} \geq \Upsilon_i((d * f) * (e * f)) \wedge \Upsilon_i(e))$$

for all $i \in I$. Thus

$$\Big(\bigwedge_{i \in I} \Upsilon_i\Big)(d) \vee \frac{1-k}{2} = \bigwedge_{i \in I} \Upsilon_i(d) \vee \frac{1-k}{2} \geq \bigwedge_{i \in I} (\Upsilon_i((d * f) * (e * f)) \wedge \Upsilon_i(e)$$

$$= \Big(\bigwedge_{i \in I} \Upsilon_i\Big)((d * f) * (e * f)) \wedge \Big(\bigwedge_{i \in I} \Upsilon_i\Big)(e)$$

Thus

$$\Big(\bigwedge_{i \in I} \Upsilon_i\Big)(d) \vee \frac{1-k}{2} \geq \Big(\bigwedge_{i \in I} \Upsilon_i\Big)((d * f) * (e * f)) \wedge \Big(\bigwedge_{i \in I} \Upsilon_i\Big)(e).$$

Hence, $\bigwedge_{i \in I} \Upsilon_i$ is an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of κ.

Theorem 4.8. The union of any family of $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p ideals of a BF-algebra κ is an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of κ.

Proof. Straightforward.

Remark 4.9. Let Υ be a fuzzy subset of a BF-algebra κ and

$$I_t = \{t \mid t \in (0, 1] \text{ such that } \Upsilon_t \text{ is a p-ideal of } \kappa\}.$$

In particular,

(1) If $I_t = (0, 1]$, then Υ is a fuzzy p-ideal of κ (Theorem 2.8).

(2) If $I_t = (\frac{1-k}{2}, 1]$, then Υ is an $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy p-ideal of κ (Theorem 4.8).

5. Conclusion

In the study of fuzzy algebraic system, we see that the fuzzy p ideals with special properties always play a fundamental role.

In this paper we define the idea of $(\overline{\alpha}, \overline{\beta})$-fuzzy p-ideal in BF-algebra and investigate some of their related properties, where $\overline{\alpha}$, $\overline{\beta}$ are any one of $\overline{\in}$, $\overline{q}_k$, $\overline{\in} \vee \overline{q}_k$, $\overline{\in} \wedge \overline{q}_k$ unless otherwise specified.

We believe that the research along this direction can be continued, and in fact, some results in this paper have already constituted a foundation for further investigation concerning the further development of fuzzy BF-algebras and their applications in other branches of algebra. In the future study of fuzzy BF-algebras, perhaps the following topics are worth to be considered:

(1) To characterize other classes of BF-algebras by using this notion;
(2) To apply this notion to some other algebraic structures;
(3) To consider these results to some possible applications in computer sciences and information systems in the future.

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